Exercise 65

Prove that cosine is a continuous function.

Solution

Start with the identity,

$$\cos a = \cos a.$$

Rewrite the left side.

$$(\sin a) \cdot 0 + (\cos a) \cdot 1 = \cos a$$

Use the formulas in (6) on page 119.

$$(\sin a) \cdot \lim_{h \to 0} \sin h + (\cos a) \cdot \lim_{h \to 0} \cos h = \cos a$$

$\sin a$ and $\cos a$ are constants and can be brought inside the respective limits.

$$\lim_{h \to 0} \sin a \sin h + \lim_{h \to 0} \cos a \cos h = \cos a$$

The limit of a sum is the sum of the limits.

$$\lim_{h \to 0} (\sin a \sin h + \cos a \cos h) = \cos a$$

Use the angle addition formula for cosine.

$$\lim_{h \to 0} \cos(h + a) = \cos a$$

Therefore, cosine is a continuous function.